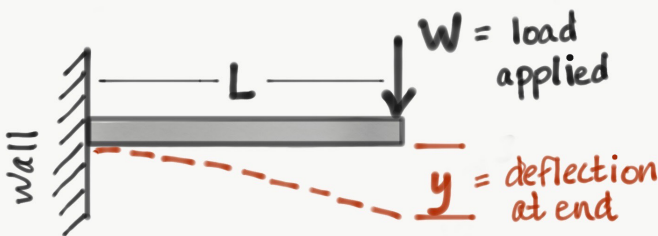


DON'T PANIC

Beam Deflection



Beam Cross-section properties: $I = \frac{b d^3}{12}$

① $b=1, d=4, I=5.3$

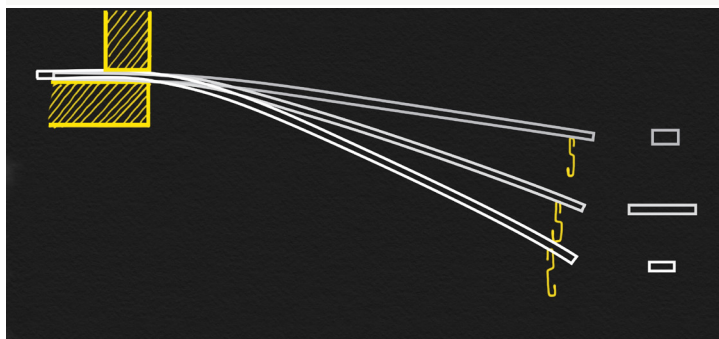
② $b=4, d=1, I=0.3$

$$y = \frac{-2WL^3}{6EI} = -\frac{2 \times W \times L \times L \times L \times 12}{6 \times E \times b \times d \times d \times d}$$

$E = \text{material property}$
 $L = \text{length}$
 $W = \text{applied load (weight)}$ } Given

$$y = -\left(\frac{2WL^3}{6E}\right) \frac{1}{I}$$

As $I \uparrow$ $y \downarrow$
 As $L \uparrow$ $y \uparrow \uparrow \uparrow$



Try to channel a little Hitchhiker's Guide to the Galaxy right now, and Don't Panic.

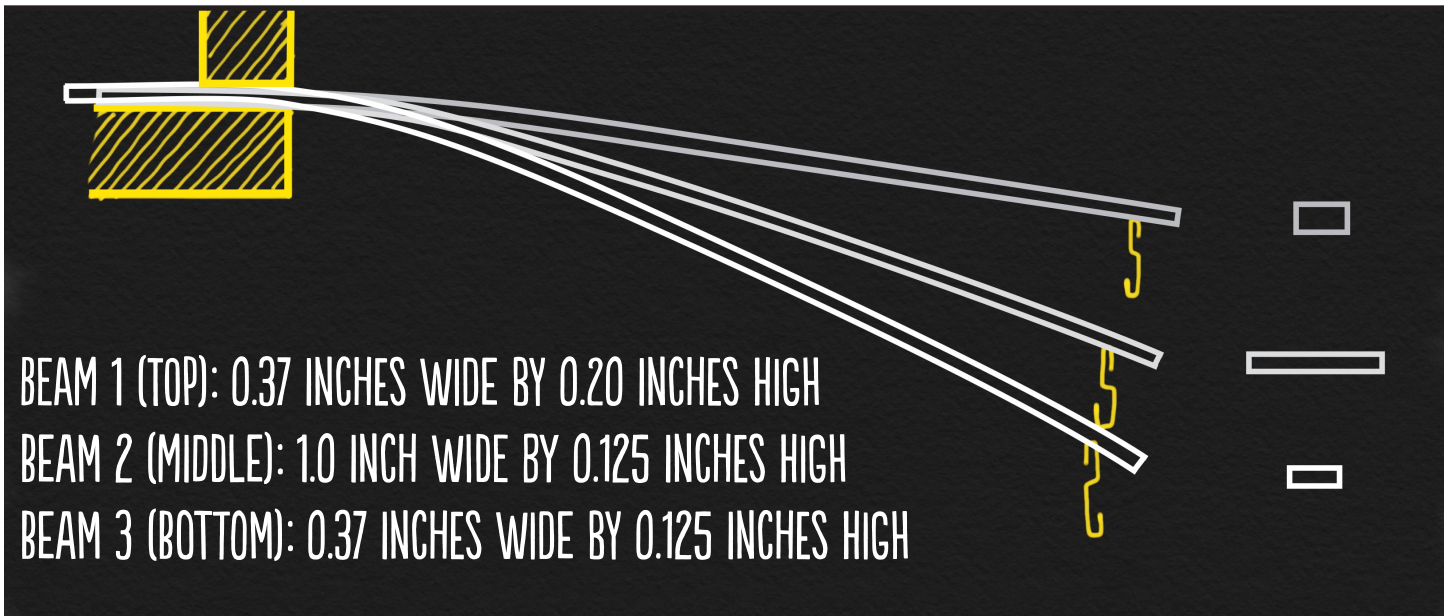
I'm going to show you a peek at the Magic of Math, and that might take you outside your comfort zone. But it's okay. You'll do fine. Promise. Just take a couple deep breaths. If you can do basic addition, subtraction, multiplication, and division with numbers, you can do this. We'll just be playing secret agents a bit, and giving code-names to some of our characters. Okay?

Supplies needed: it'll be easier if you follow along with a demonstration. All you need is a plastic knife, or a thin wood paint stir stick. Or something thin and long and a bit flexible. Got it?

First take your plastic knife with the flat side down. Use your hand to clamp the handle end to the end of a table, with the blade pointing out. Now take your finger and push down on the end of the blade. What do you notice? It bends pretty easily, right?

Now, flip the knife and hold the narrow edge of the handle against the table, and carefully push down on the end of the blade with your fingers. You might want to grip either side of the blade and push down. What do you notice this time? It's pretty stiff right? Doesn't bend nearly as much.

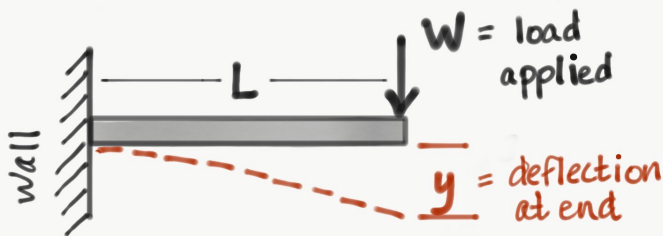
The hairy scary equation at left is a formula that engineers use to predict how things bend. Once you get to know it, it even tells you what things are most important if you want to make something stiffer, or more flexible. Pretty cool, huh?



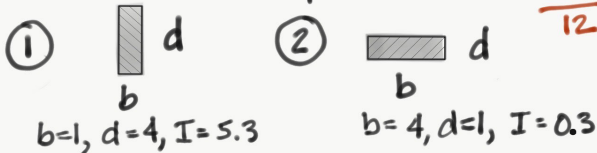
BEAM 1 (TOP): 0.37 INCHES WIDE BY 0.20 INCHES HIGH
 BEAM 2 (MIDDLE): 1.0 INCH WIDE BY 0.125 INCHES HIGH
 BEAM 3 (BOTTOM): 0.37 INCHES WIDE BY 0.125 INCHES HIGH

SKETCH OF BEAM BENDING EXPERIMENT RESULTS USING STRIPS OF BALSAMIC WOOD, EACH 36 INCHES LONG. A PLASTIC BAG WITH FOUR QUARTERS FOR WEIGHT WAS HUNG OFF THE END OF EACH BEAM USING A PAPER CLIP.

Beam Deflection



Beam Cross-section properties: $I = \frac{b d^3}{12}$



$$y = \frac{-2WL^3}{6EI} = \frac{-2 \times W \times L \times L \times L \times 12}{6 \times E \times b \times d \times d \times d}$$

$E = \text{material property}$
 $L = \text{length}$
 $W = \text{applied load (weight)}$ } Given

$$y = -\left(\frac{2WL^3}{6E}\right) \frac{1}{I}$$

As $I \uparrow$ $y \downarrow$
 As $L \uparrow$ $y \uparrow \uparrow \uparrow$

Okay, still with me? let's take a closer look at those not-so-hairy-scary equations, and try some simulated experiments.

For this experiment, I took three thin and long wood beams, like you can buy in a craft store. I hung a weight off the end, clamped the other end to the table, took a picture of the result. The sketch above shows the results. What do you notice?

Beams 2 and 3 are the same thickness, but beam 2 is wider. It doesn't bend as much.

Beams 1 and 3 are the same width, but beam 1 is a bit thicker. It bends the least.

So, beam **thickness** and **width** matter.

Can we predict these results using our equation? Yes, we can!

Y represents how much the beam bends at the end. It depends on the **weight**, W , the **length**, L (really important—it gets multiplied by itself 3 times!), a **material constant**, E , and a property of the cross-section, I , called the **moment of inertia**. If we look at the equation for I , we see that d , the beam thickness, is multiplied by itself 3 times. Like L , it has a strong influence on stiffness.

