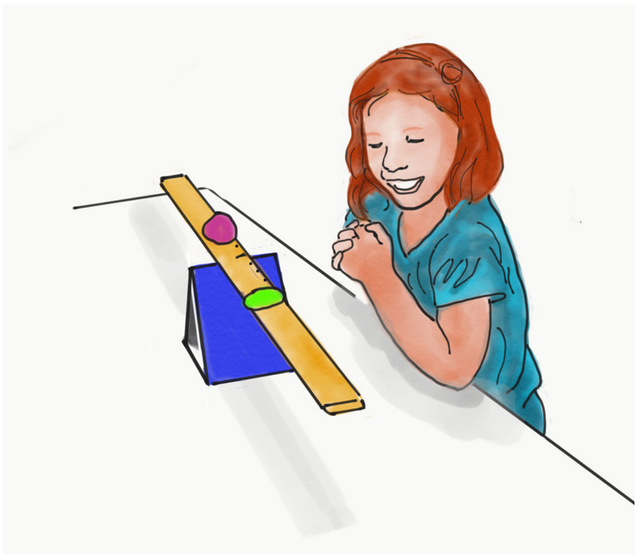
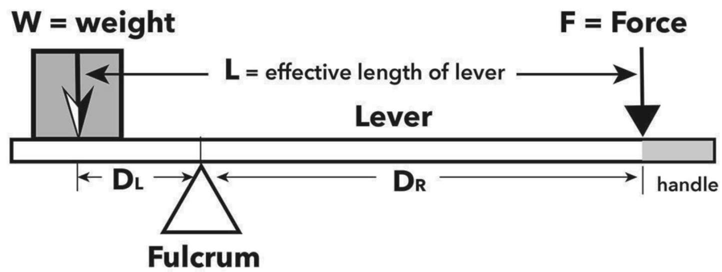
EXPLORING MECHANICAL ADVANTAGE & THE LEVER LAW

Give me a lever long enough and a fulcrum on which to place it, and I shall move the world.
— Archimedes





From teeter-totters, to car jacks, to huge cranes used to lift heavy objects and build skyscrapers, the lever is one of the most basic and used mechanical devices. You see them used every day, in a wide range of situations.

The hole punch uses mechanical advantage and leverage to cut holes in paper. A staple remover uses leverage to pull out staples. A stapler uses leverage to multiply the force applied to push staples into paper.

The list goes on and on.

But how exactly does this work? Let's do a few experiments to find out.

Lesson Objectives

- Develop concrete understanding of mechanical advantage, and be able to explain how it works
- Through experiments, deduce the Lever Law relationship
- Use the Lever Law to predict behavior and as a mechanical calculator
- Examine the special case of the Lever Law where the distance from the fulcrum is the same on both side, allowing the Lever Law to function as a scale
- Demonstrate how we use the Lever Law to help with balance

Materials Used

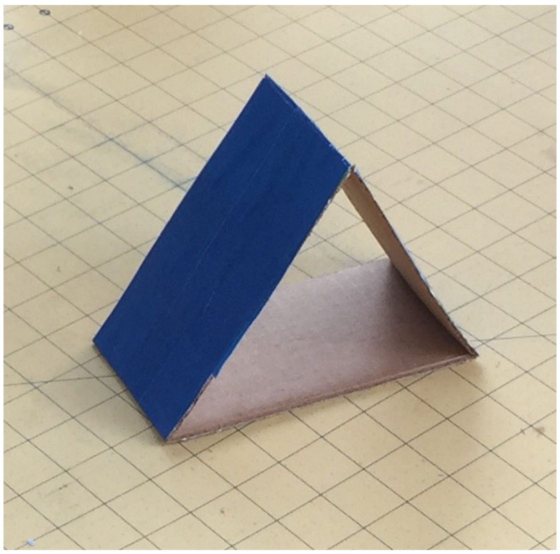
- Wood yardstick – with inches marked off from center
- Fulcrum – made from cardboard & duct tape
- Play dough – pre-measured in 1, 2, 3, 4, and 5 ounce balls
- Digital kitchen scale
- Paper and pencil to record results (data sheets provided)

Preparation

You can tag team the preparation with your daughter. Have her help build the cardboard fulcrum and make – or at least knead, measure, and shape – the play dough balls! Sometimes the prep is half the fun. You also may want her to help you do the yardstick markings.

The Yardstick

You can use any yardstick you have on hand. If you need to buy one, check Lowe's or a similar home improvement store for an inexpensive, unfinished one. Lowe's had basic wood yardsticks in their paint department for 98 cents. They were a bit rough, so you might want to sand the edges. The best part is that there is room to add your own markings. You can use a fine point Sharpie permanent markers in two contrasting colors (like blue and orange) to make markings. Make a thick blue line at the midpoint of the yardstick where the fulcrum would be placed. Add thin blue lines at 5-in. intervals, marking from the center out. Add thin orange lines at the remaining 1-in. intervals. Finally, mark the inches from the center out using the same color as the lines. This makes it easier to read the distance from the fulcrum.



The Fulcrum

All you need for a fulcrum is a piece of cardboard folded and taped into a triangular cross-section instead of a square or rectangular cross-section. The exact dimensions don't matter, just make it a convenient height for your daughter to work with. The example above was made from three pieces of 3 by 6 in. cardboard cut from the flaps of a box then taped together with duct tape.

Fulcrum Dimensions

Side lengths between 6 and 7 inches work well. Your fulcrum will be tall enough to get your hands easily under the yardstick to support it while changing the weights, but not so tall the angle gets steep and the play dough falls off.

Side widths of 3 inches or wider work well. Your fulcrum needs to be wider than your yardstick to provide support. An Amazon A1 box has 4 flaps that are about 3.5" wide by 6.5" long. That would work well.

Note: Be sure that your flat sides do not have a fold in them. Folds act as hinges, and you won't have a triangular structure anymore. You will have a mechanism.

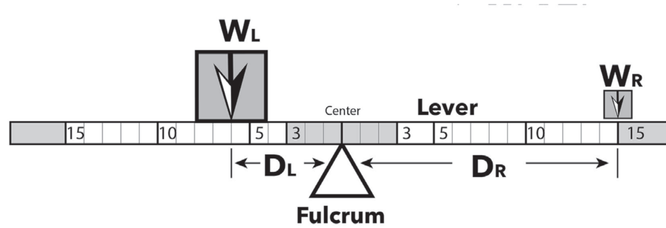
Play Dough

Make or buy play dough. Shape it into balls in the following weights: 1 ounce, 2 ounces, 3 ounces, 4 ounces, 5 ounces. If you're metric system based, use similar ratios such as 10 grams, 20 grams, 30 grams, 40 grams, 50 grams. This will help make the experiment (and the math) simpler. This experiment uses play dough because it's easy to shape into desired weights, and you can "smush" it onto a yardstick so that it stays in place and doesn't slide off.

The Data Sheet

You can use the data sheet (PDF resource provided), or make your own table on a sheet of paper (see example below). Your daughter will need to record the results from each test. You'll need 7 columns to record the results:

1. Test number
2. W_L, the weight used on left hand side
3. D_L, the distance from fulcrum on the left-hand side
4. The product of the weight x distance on the left-hand side
5. W_R, the weight used on the right-hand side
6. D_R, the distance from fulcrum on the right-hand side
7. The product of weight x distance on the right-hand side.



Test #	W_L	D_L	$W_L \times D_L$	W_R	D_R	$W_R \times D_R$
1						
2						
3						
4						

EXPLORING MECHANICAL ADVANTAGE & THE LEVER LAW

Keep the Math Simple: Round to Whole Numbers

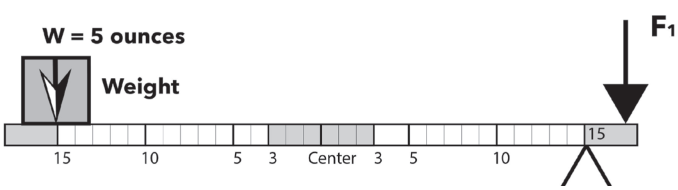
To make the math simple, we're going to work in whole numbers.

- **Round off weights to the nearest ounce**. The digital scale may jump from .99 to 1.02 oz. We'll call it an even ounce. That's close enough.
- **Measure from the center of the play dough balls, and place balls on even inch increments from the fulcrum.**

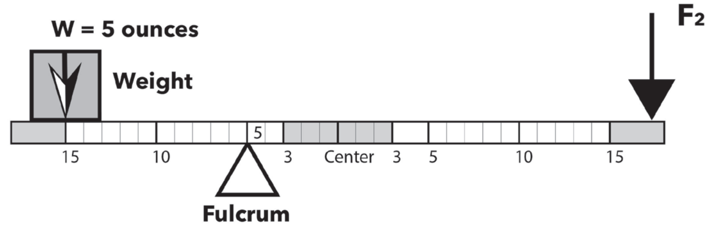
Note that the play dough balls may be an inch or more wide. But, by measuring the distance from the center of each ball to the fulcrum, we'll get a good average estimate of where that weight acts. It will simplify our math to choose to place the center the play dough balls on even inch increments from the fulcrum, so we'll help the girls in Step 2 by telling them where to place the heavy balls so the math works out.

The Experiment

Step 1 – Experiencing Mechanical Advantage
You don't have to follow these steps exactly. The examples provided use conditions that will give you a feel for very different results. Feel free to experiment!



Step 1a
Place the 5 oz. ball 15 in. to the left of center on the yardstick.
Place the cardboard fulcrum 15 in. to the right of center on the yardstick (marked with an arrow and labeled F_1 for Force 1).
Use your hand to press down about 17 in. to the right of center to lift the weight.
How does it feel? Is it difficult or easy?



Step 1b
Keep the 5 oz. ball of play dough at the same location.
Move the cardboard fulcrum 5 in. to the left of center on the yardstick.
Use your hand to press down about 17 in. to the right of center to lift the weight (place marked with an arrow and labeled F_2 for Force 2).
How does it feel? Is it more difficult or easier than before? Did you have to use more effort (or force) in the first part or the last part of this experiment?

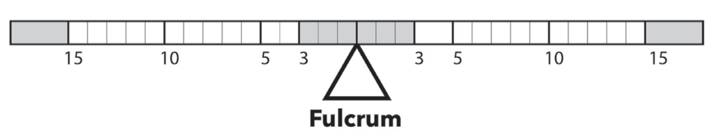
This is mechanical advantage. **The closer the weight is to the fulcrum, and the farther away the balancing force (your finger) is from the fulcrum - longer lever - the easier it is to lift the weight.**

Step 2 – Deducing the Lever Law

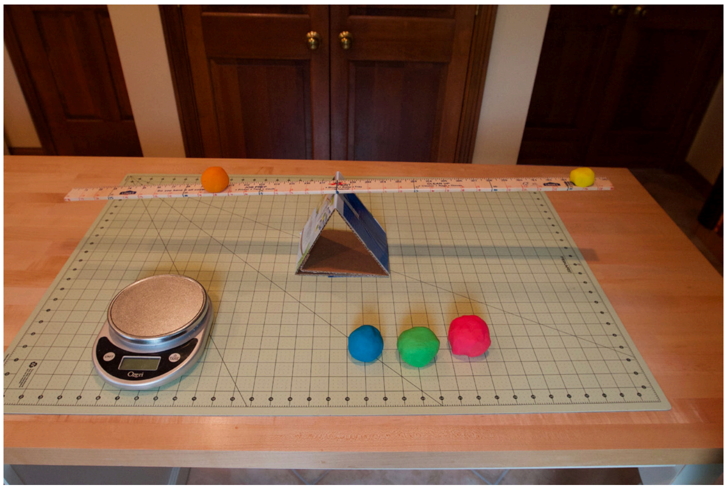
We will test 2, 3, 4, and 5 oz. balls of play dough placed 3 in. to the left of the fulcrum and see where we need to place a 1 oz. ball of play dough on the right side of the fulcrum to achieve balance.

First, however, we need to see what balance looks like with our yardstick and fulcrum.

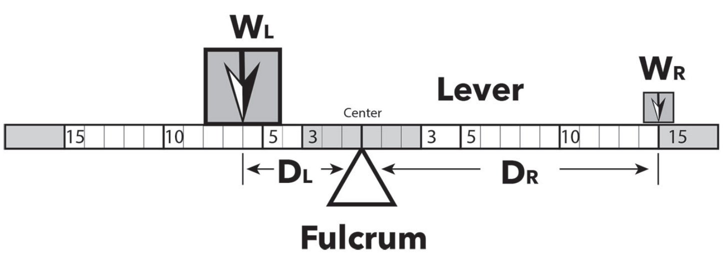
Step 2a–Checking Balance–yardstick & fulcrum alone
Support the yardstick from underneath, with one hand on each side of the fulcrum. Center the yardstick on the fulcrum at the marked balance.
When it is in position, gently lower your hands away from the yardstick.
If the yardstick has been centered well, there should be a moment of hesitation when it balances with both ends off the ground before one end sinks. Eventually an air current will tip the balance.



Step 2b – Testing the Lever Law



To simplify the experiment, place the test play dough ball (2, 3, 4, or 5 oz.) at the same location 3 in. to the left of the fulcrum. Remember to center the balls at the 3-in. mark.



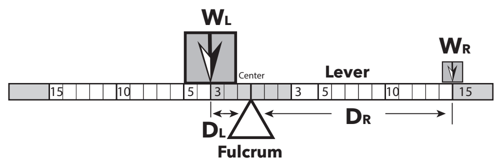
Terms:
W = Weight
D = Distance to Fulcrum

Subscripts:
L = Left side of Fulcrum
R = Right side of Fulcrum

Our objective is to figure out where to place the 1 oz. ball of play dough to achieve balance for each test.
Once we achieve balance – that moment of hesitation where both ends of the yardstick are in the air – note the distance that the center of the 1 oz. ball of play dough is away from the fulcrum.
Enter the distance in the data table and calculate the product of weight time distance.

Exploring Mechanical Advantage & The Lever Law

"Give me a lever long enough and a fulcrum on which to place it, and I shall move the world." – Archimedes



Terms:
W=Weight
D=Distance to Fulcrum
Subscripts:
L=Left side of Fulcrum
R=Right side of Fulcrum

Test #	W_L	D_L	$W_L \times D_L$	W_R	D_R	$W_R \times D_R$
1	2	3	6	1		
2	3	3	9	1		
3	4	3	12	1		
4	5	3	15	1		

What do you notice? Are you seeing a trend? This is the Lever Law. It can be written as:

$$W_L \times D_L = W_R \times D_R$$

The fulcrum represents the "equal" sign. We are doing physical multiplication. As long as the yard-stick is in balance on the fulcrum, the product of the weight and distance from the fulcrum on the left side must equal the product of the weight and distance from the fulcrum on the right side.

In this case, the lever length is the distance between the centers of the two weights. The longer the lever length, and the closer the fulcrum is to the heavier weight, the more mechanical advantage we have.

Step 3 - Using the Lever Law as a Mechanical Calculator

You should have noticed that the product of the weight times the distance from the fulcrum on the left side equals the product of the weight times the fulcrum on the right side.

So, let's mix it up a bit. Think of some combina-tions of weight times distance that are less than or equal to 16. We have weights of 1 to 5 oz. Think of a combination, and test it out!

Hint: You can place a smaller ball of dough on top of a larger one to increase the total weight.

Note: If your results don't appear to come out correctly, check your weights! You might have mixed up a 3-oz. ball with a 2-oz. ball, and the product you think you have will be different from the one you really have.

Step 4 - Using the Lever Law as a Scale

A special case of the lever law is where you keep both distances from the fulcrum the same.

$$W_L \times D_L = W_R \times D_R \text{ simplifies to } W_L = W_R .$$

Mechanical kitchen scales use this principle. Using a set of "standard" or known weights, you can measure out an equivalent weight of flour or other ingredient. When the scale balances, you know you have the desired weight.

If desired, you can test some combinations of weights at equal distances from the fulcrum, say 10 inches. You can "stack" balls of play dough to get an equivalent weight. 1+2= 3, 2+3=5.

Summary

- What did you learn about the Lever Law?
- How could you make it easier to lift a heavy weight?
- How can you use the lever law as a calculator?
- How can you use the lever law as a scale?
- How else can you think to use the Lever Law?

The longer the lever, and the closer the heavy weight is to the fulcrum, the greater the mechanical advantage. You can use the Lever Law to calculate how to balance two weights, or to figure out how much force (or weight or effort) you need to apply to lift something.

A simplified form of the Lever Law can be used as a balance scale. Some kitchen scales use this principle.

If you've gone to the playground and played on the teeter-totter, you've used the lever law! What was fun about the teeter-totter? For me, it was that moment of weightlessness you experience when your side is at the maximum height, just before you start coming down. That's what Stuart Brown calls gravity play.

To maximize that moment of weightlessness, it helps to have people of nearly equal weights on the teeter-totter, because you'll have an extended moment of hesitation when it balances. If the weights are very lopsided (one parent, one child), the teeter-totter is more jarring and just wants to plop down to the heavier side.

Now that you've learned about the lever law, can you think how to better balance a heavy adult on a teeter-totter? Try it out next time you're at at a park with a teeter-totter. (Hint: move the heavier person closer to the fulcrum! Be safe though!)

Another use of the Lever Law is to assist with balance. Have you ever tried to balance on one foot? What do you do when you start to teeter? If I stumble while walking, I throw out both arms to help catch my balance. If I'm balancing on one foot and start to lean in one direction, I throw out an arm or leg in the opposite direction to balance. Your body knows the lever law and already uses it.

If you've ever been to the circus (or watched one on TV), perhaps you've seen a tightrope walker with a long pole. That long pole has a distributed weight and a very long lever arm. When it is carried at its balance point, it helps a lot with balance. Try it out! You can test the effect of lever length with two equal weighs with a yard stick and a ruler. Or even just try balancing the ruler or yardstick on their own. Then try adding a weight at the center of the yardstick or ruler just over the fulcrum. Which is easier to balance for longer? Try different weights out. Does it make a difference with 1 ounce weight versus 2, 3, 4, or 5 oz.? My yardstick weighs just over 3 oz. Does it make a difference if the weight is less than or equal to the weight of the stick?

Can you think of any other ways you've used the Lever Law?

Can you think of any other experiments you'd like to try out? Go for it!

Congratulations! You've completed the Lever Law experiment!

Resources

Play dough - recipe given below, or buy Play Doh
Wood yardstick
Digital kitchen scale

The Best Playdough Recipe -

ingredients from
http://tinkerlab.com/rainbow-play-dough/

Ingredients:

2.5 cups flour
1 1/4 cup salt
1 1/2 tbsp. cream of tartar
5 tbsp. vegetable oil
2.5 cups water
Food coloring

Marsha's Instructions:

Mix flour, cream of tartar, salt and vegetable oil in a large bowl until well mixed. It'll be crumbly. Gradually add in the water until somewhat smooth. Don't worry about lumps.

Transfer mixture into a large saucepan. Cook over a low heat, stirring frequently. The mixture will gradually become less soupy and more dough-like. Pinch a piece of dough periodically to check consistency. When you're happy with the consistency (not gooey, dough on edges of pan starting to look dry), remove from heat and transfer dough to a washable cutting mat (or other surface that will not be permanently stained from using food coloring).

Knead the dough to even out any remaining lumps, then divide it into the desired number of pieces for coloring. If you need specific weights for each color (for example, lever law experiment and you want each ball to be a different color), use a kitchen scale to divide the dough into the desired weights before adding color.

NOTE: food coloring will stain your hands, and may take a couple days to wear off. You may wish to wear latex or vinyl gloves while kneading in the food coloring!

To add color, flatten ball slightly, create a dimple in the center and place a drop of food coloring in it.

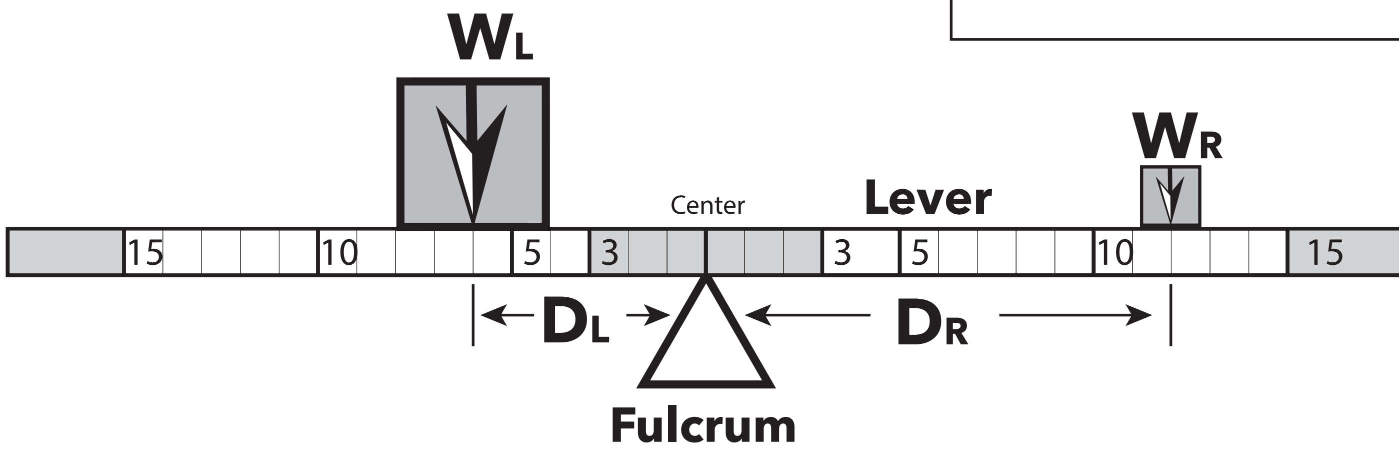
Fold edges around the drop of color and knead the dough until the coloring is worked through. If you don't quite have the color you want, add more. So - start with less than you think you need and add more until you get what you want.

If you want a lot of the same color, mix it in one batch. It is hard to duplicate shades exactly between batches.

Use the dough right away or store it in a ziplock bag or similar sealable bag, pressing the air out as much as possible to prevent it drying out. Unused it will keep for months.

The Lever Law as a Mechanical Calculator

$$W_L \times D_L = W_R \times D_R$$



Test #	W_L	D_L	$W_L \times D_L$	W_R	D_R	$W_R \times D_R$
1						
2						
3						
4						