

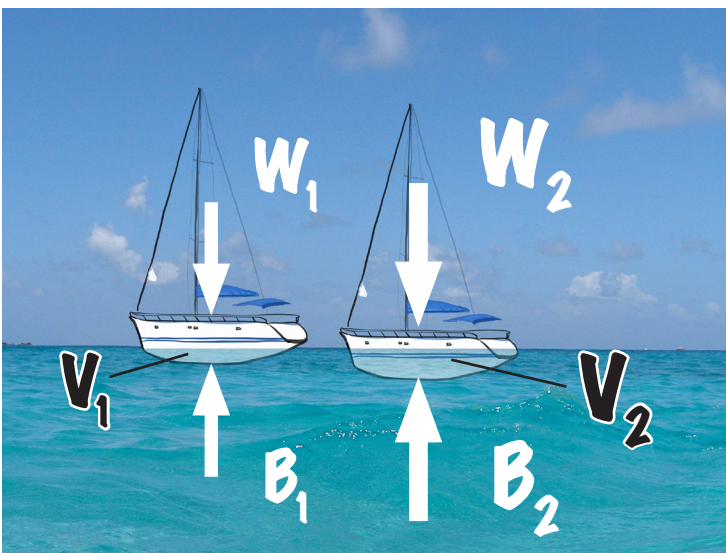
Force balance

Weight is a **force** or **load** that is often modeled as acting on a **point**.

If an object isn't moving, then the forces acting on it must be in balance.

The sailboat at left is resting at anchor. The **weight** of the boat, **W**, is supported by the **buoyant force** of the water, **B**.

$$W = B$$



Fluids and Buoyancy

Solids hold their own **shape** and **volume**.

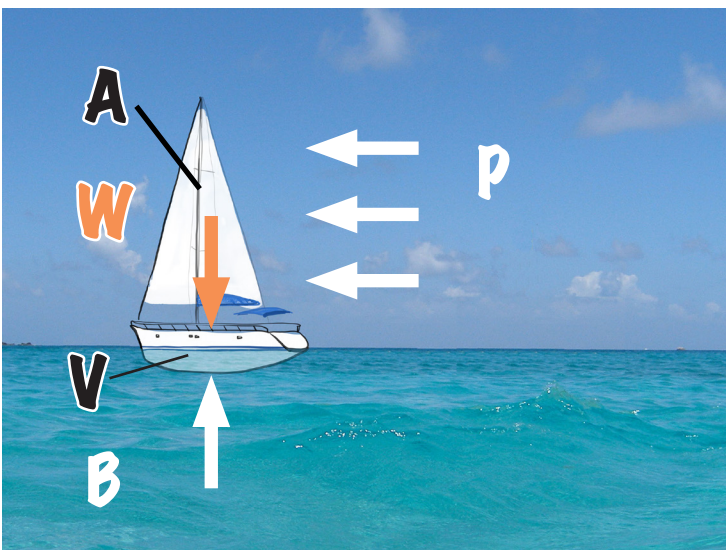
Fluids—*liquids and gases*—have **no fixed shape** and yield easily to pressure.

Liquids—like water—have a **constant volume**.

Gases—like air—will **expand freely** to fill a container, having **no fixed shape**, and **no fixed volume**.

The **weight** of the boat, **W**, causes a **volume** of water to be displaced, **V**, which depends on the **density** of the water, ρ . This is the buoyant force, $B = \rho \cdot V$.

We can tell that the boat on the right is heavier than the boat on the left, because the volume of water displaced is larger. $V_2 > V_1$, therefore $W_2 > W_1$.



Pressure—a distributed load

Pressure, **P**, is a force exerted over an **area**, **A**, like wind hitting the sail of a sailboat.

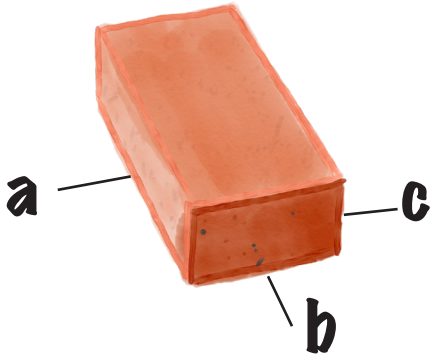
We can calculate an equivalent force by multiplying the pressure by the area on which it acts:

$$F = P \cdot A$$

The sailboat, left, will move to the left because of wind pressure on the sail, unless there is a resisting force to balance it.

Note: this handout uses a dot (•) to indicate multiplication, because x is often used as an axis direction, or a variable.

$$V = a \cdot b \cdot c$$



Volume

Solids hold their own shape and volume.

Volume can be measured or calculated different ways. For **rectangular solids**, like this brick, there's a simple formula—you just need to know the length of each side:

$$V = a \cdot b \cdot c$$

So, a brick 12 inches long by 4 inches wide and 2 inches high would have a volume, V:

$$V = 12 \cdot 4 \cdot 2 = 96 \text{ cubic inches, or } 96 \text{ in}^3$$

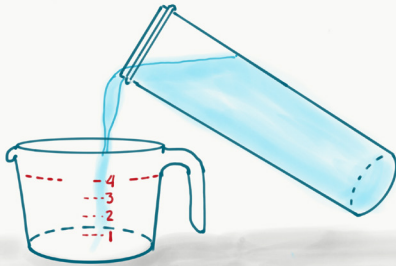
Scientists keep track of the units of measurement—such as inches, feet, or meters. This makes it easier to use conversion factors to translate to a more convenient system of units if needed.

Liquids have a constant volume

If we want to measure the **volume of a liquid**, or measure out a certain amount of a liquid, we can use a calibrated measuring cup.

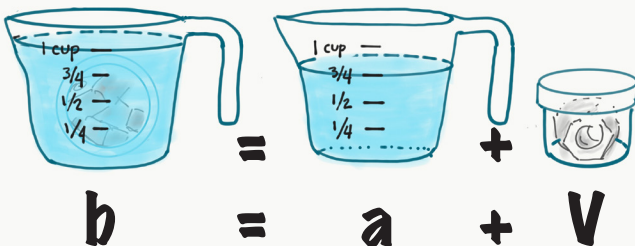
This makes use of the property of liquids that they take the shape of their container.

This also gives us a clever way to measure the volume of an irregular solid, as we'll see below.



$$b = a + V \quad (\text{conservation of mass})$$

$$V = b - a \quad (\text{measure } a, b, \text{ solve for } V)$$



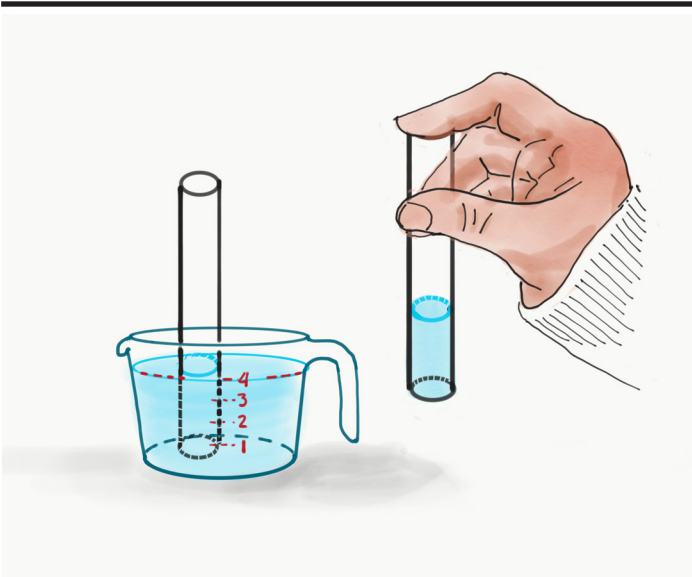
Volume calculation by liquid displacement

In the sketch at left, we want to figure out the volume of the plastic container of hex nuts (far right).

Place the container of hex nuts in the measuring cup, then fill it with water to a known level (1 cup in this example)—figure at far left.

Next, take out the container of hex nuts and measure the level of the water remaining (3/4 cup in this example). We now know the value of a and c, and can solve for b, using conservation of mass (no matter was created or destroyed during this experiment):

$$V = b - a = 1 \text{ cup} - 3/4 \text{ cup} = 1/4 \text{ cup}$$



So what happens when we put a straw in a container of water, put our finger over the top, and lift the straw out?

Well, as long as you have a good seal on the top of the straw, you should be able to lift out a column of water.

But why?

Isn't water **heavier** than air? Isn't that why rain falls to the ground, and lakes don't float in the air?

Something has to be holding the water up in the straw. Can you guess what it is?

It's air pressure.

Air is all around us, pressing down at sea level at a pressure of about 14.7 pounds per square inch (lb/in²).

Area and volume

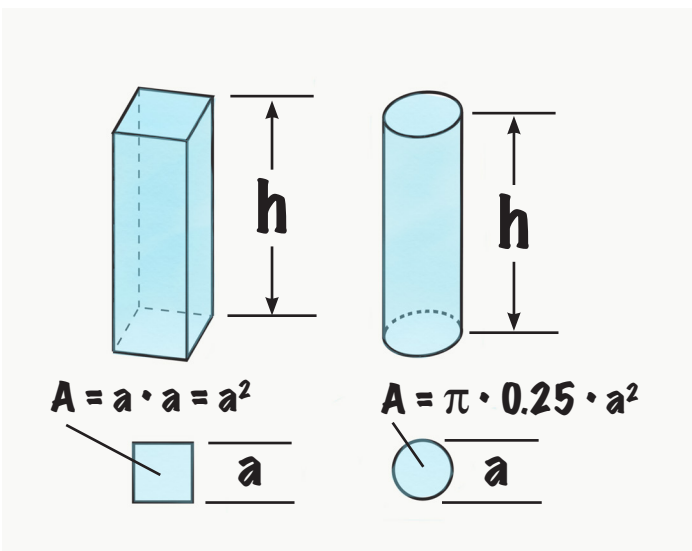
The volume of a **column** is equal to the area of its cross-section, A , times the height of the column, h .

$$V = A \cdot h$$

Different shapes have different areas, but don't worry about that. The areas will cancel out in the force balance.

The **weight** of a column of water is equal to its volume multiplied by the density of the water:

$$W_{\text{water}} = \rho \cdot V = \rho_{\text{water}} \cdot A \cdot h$$



Force balance on the straw

Let's look at the forces acting on the water in the straw. We can calculate the **force** due to **air pressure**:

$$F_{\text{air}} = P \cdot A = 14.7 \cdot A \quad (\text{pounds})$$

We can calculate the **weight** of the **water**:

$$W_{\text{water}} = \rho_{\text{water}} \cdot A \cdot h = 0.036 \cdot A \cdot h \quad (\text{pounds})$$

Guess what? Now we can solve to find the **height** of the column of water needed to offset the pressure of the air:

$$0.036 \cdot A \cdot h = 14.7 \cdot A$$

$$h = 407 \text{ inches, or just under 34 feet for tap water}$$

If we take our finger off the top of the straw, the forces from the air pressure cancel out, so the water will run out.

